POD

Q1: Are all circles similar?

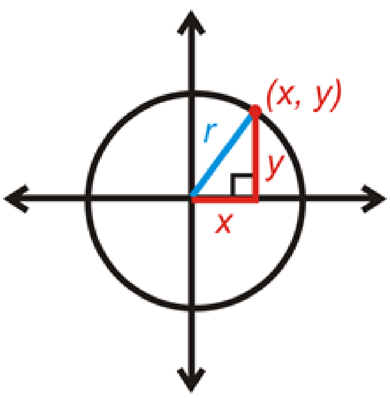
Q2: The equation of a circle is

$(x-h)^2 + (y-k)^2 = r^2$

What do the letters h and k represent?

Lesson objectives: learn the properties of circles in the coordinate plane and apply them to solve problems.

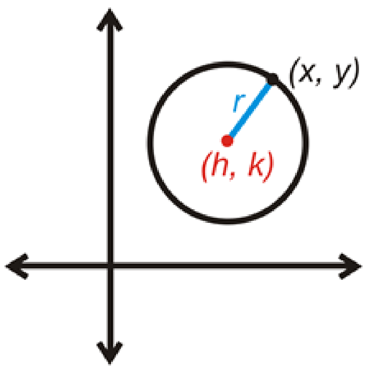
Recall that **a circle is the set of all points in a plane that are the same distance from the center**. This definition can be used to find an equation of a circle.



Let's start with the circle centered at (0, 0). If (x,y) is a point on the circle, then the distance from the center to this point would be the radius, r. x is the horizontal distance and y is the vertical distance. This forms a right triangle. From the **Pythagorean Theorem**, the equation of a circle ***centered at the origin*** is: $x^2+y^2=r^2$.

The center does not always have to be on (0, 0). If it is not, then we label the center (h,k). We would then use the **Distance Formula** to find the length of the radius. Given any two points (x_1,y_1) and (x_2,y_2), the distance between them is:

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$



$$d=r=\sqrt{(x-h)^2+(y-k)^2}$$

For h=3, k=-2

$r=\sqrt{(x-3)^2+(y-(-2))^2}$

For x=1, y=4

$r=\sqrt{(1-3)^2+(4+2)^2}$

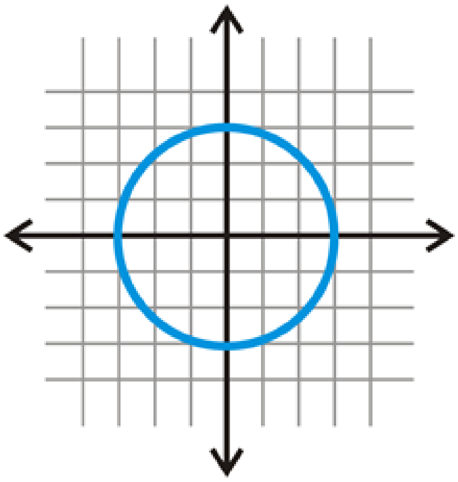
$r=\sqrt{(-2)^2+(6)^2}=\sqrt{4+36}$

$r=\sqrt{40}=2\sqrt{10}$

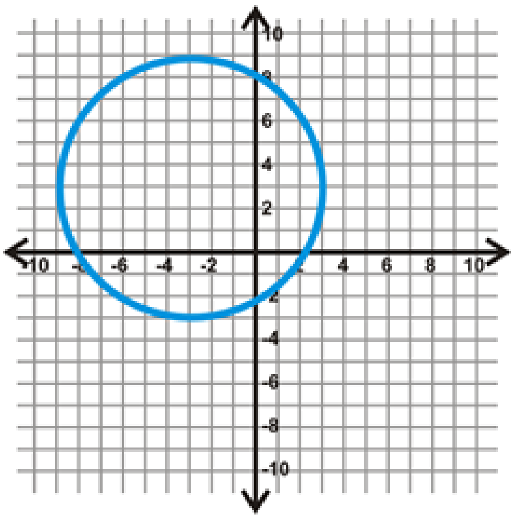
If you square both sides of this equation, then you would have the standard equation of a circle. **The standard equation of a circle with center (h,k) and radius r is $r^2=(x-h)^2+(y-k)^2$.**

Example A Graph $x^2+y^2=9$.

The **center** is (0, 0). Its **radius** is the square root of 9, or 3. Plot the center, plot the points that are 3 units to the right, left, up, and down from the center and then connect these four points to form a circle.



Example B Find the equation of the circle below.



First locate the center. Draw in the horizontal and vertical diameters to see where they intersect.

Plugging this into the equation of a circle, we get:

$(x-(-3))^2+(y-3)^2=6^2$

or

$(x+3)^2+(y-3)^2=36$

Example C Determine if the following points are on $(x+1)^2+(y-5)^2=50$.

a) (8, -3) b) (-2, -2)

Procedure: first, plug in the points for x=8 and y=-3 in to the equation of the circle: $(x+1)^2+(y-5)^2=50$.

a) $(8+1)^2+(-3-5)^2 \stackrel{?}{=} 50$

$9^2+(-8)^2 \stackrel{?}{=} 50$

$81+64 \stackrel{?}{=} 50$

$145 \neq 50$

thus, (8, -3) is *not* on the circle

Next, plug in the points for x=-2 and y=-2 in to the equation of the circle: $(x+1)^2+(y-5)^2=50$.

b) $(-2+1)^2+(-2-5)^2 \stackrel{?}{=} 50$

$(-1)^2+(-7)^2 \stackrel{?}{=} 50$

$1+49 \stackrel{?}{=} 50$

$50 = 50$

thus, (-2, -2) *is* on the circle

Do now!

Find the center and radius of the following circles.

1. $(x-3)^2+(y-1)^2=25$

2. $(x+2)^2+(y-5)^2=49$

3. Find the equation of the circle with center (4, -1) and which passes through (-1, 2).

Answers:

1. Rewrite the equation as $(x-3)^2+(y-1)^2=5^2$. The center is (3, 1) and $r=5$.

2. Rewrite the equation as $(x-(-2))^2+(y-5)^2=7^2$. The center is (-2, 5) and $r=7$.

Keep in mind that, due to the minus signs in the formula, the coordinates of the center have the ***opposite signs*** of what they may initially appear to be.

3. First plug in the *center* to the standard equation.

$(x-4)^2+(y-(-1))^2 = r^2$

$(x-4)^2+(y+1)^2 = r^2$

Now, plug in (-1, 2) for x and y and solve for r.

$(-1-4)^2+(2+1)^2 = r^2$

$(-5)^2+(3)^2 = r^2$

$25+9 = r^2$

$34 = r^2$

Substituting in 34 for r^2, the equation is $(x-4)^2+(y+1)^2=34$.

Exit Slip!

Find the *center* and *radius* of each circle.
Then, *graph one circle*.

1. $(x+5)^2+(y-3)^2 = 16$
2. $x^2+(y+8)^2 = 4$
3. $(x-7)^2+(y-10)^2 = 20$
4. $(x+2)^2+y^2 = 8$